Surface Area and Volume

Surface Area of Prisms

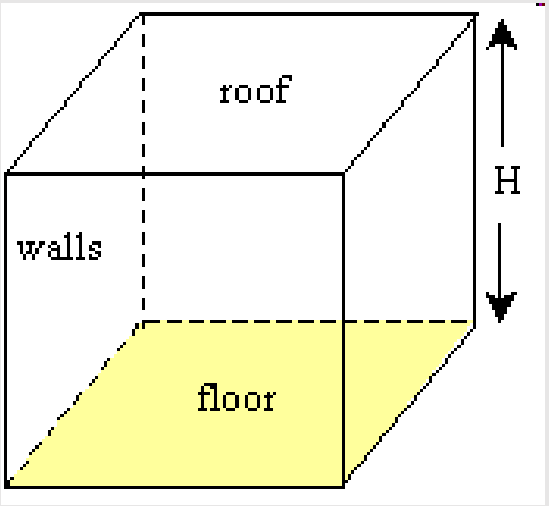
Surface Area = The total area of the surface of a three-dimensional object

(Or think of it as the amount of paper you'll need to wrap the shape.)

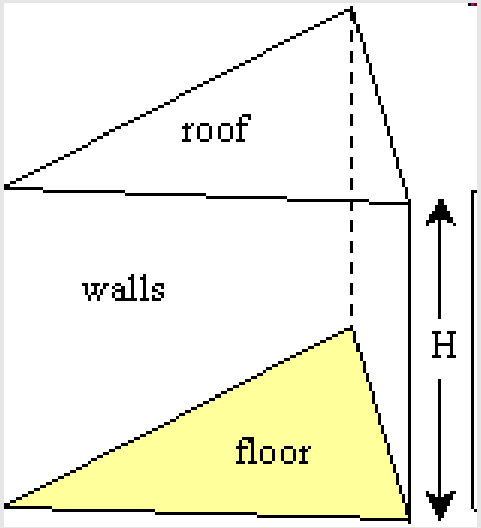
Prism = A solid object that has two identical ends and all flat sides.

We will start with 2 prisms – a rectangular prism and a triangular prism.

Rectangular Prism

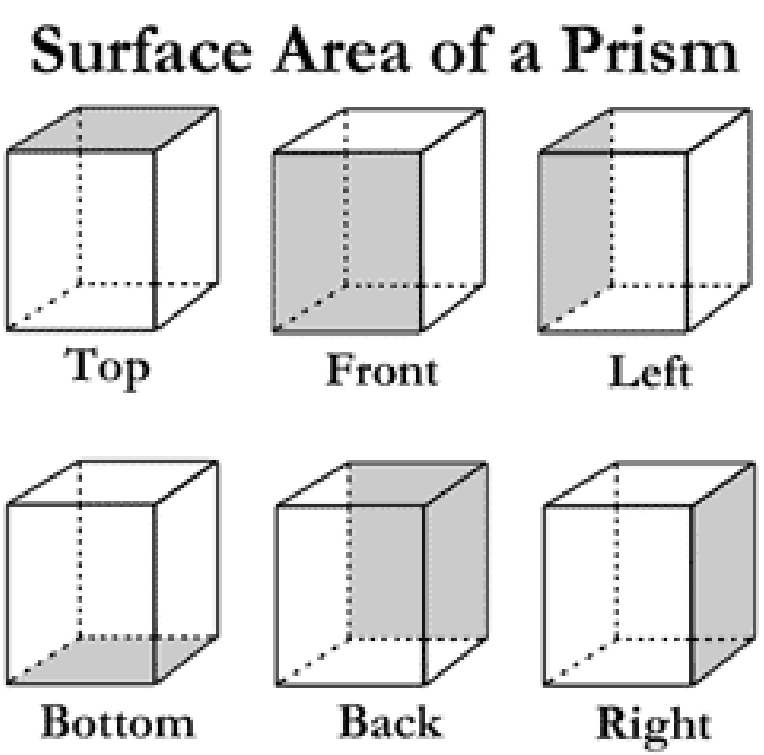


Triangular Prism

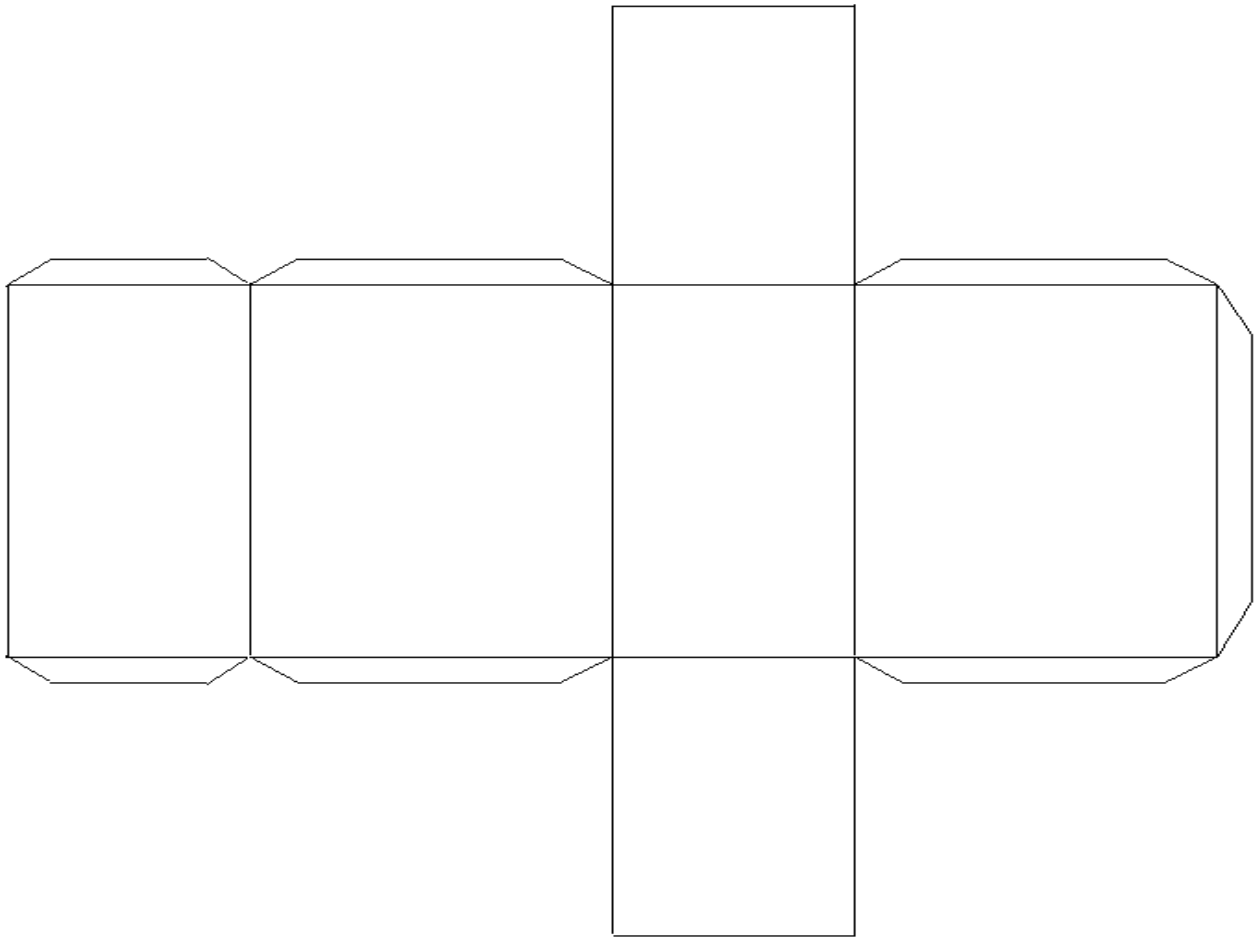


Surface Area (SA) of a Rectangular Prism

Like dice, there are six sides (or 3 pairs of sides)



Prism net - unfolded



Rectangular Prism

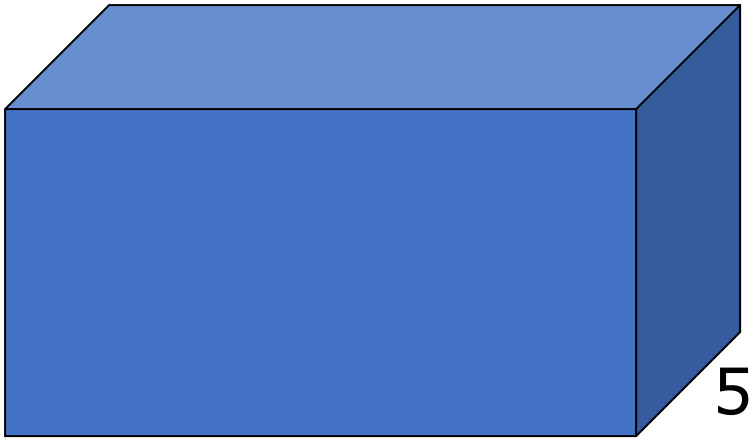
- Add the area of all 6 sides to find the Surface Area.

6 - height

5 - width

10 - length

$$SA = 2lw + 2lh + 2wh$$



6 - height

5 - width

10 - length

SA = 2lw + 2lh + 2wh

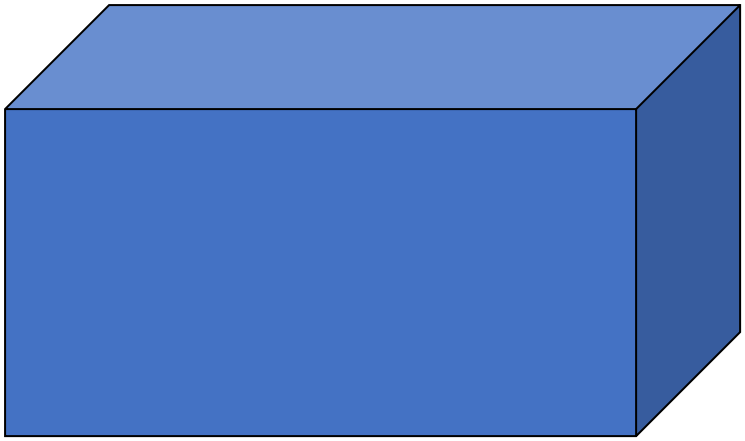
SA = 2 (10 x 5) + 2 (10 x 6) + 2 (5 x 6)

= 2 (50) + 2(60) + 2(30)

= 100 + 120 + 60

= 280 units squared

Practice



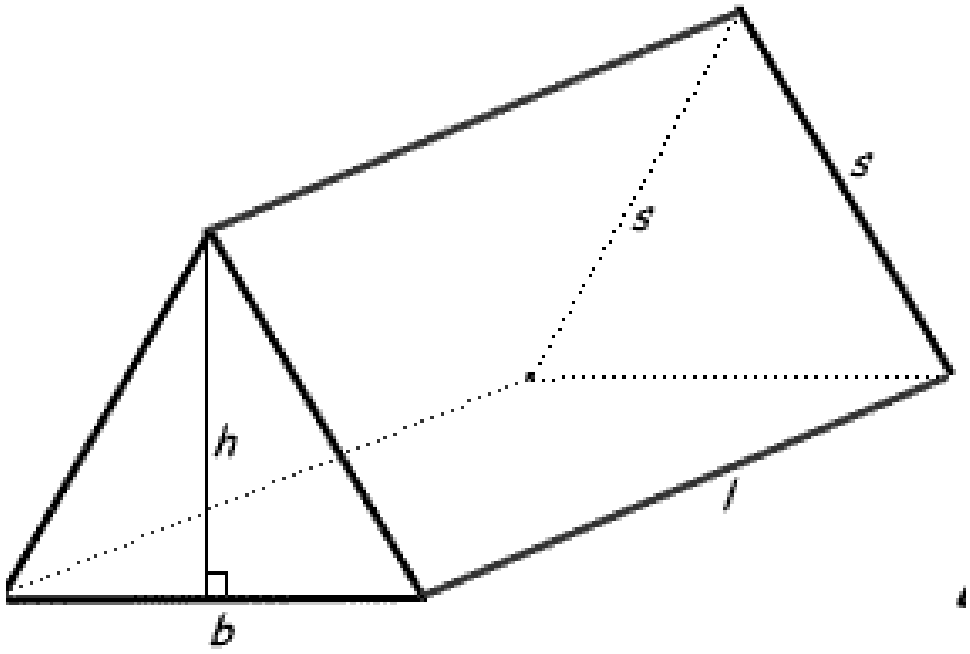
SA = 2lw + 2lh + 2wh
= 2(22 x 10) + 2(22 x 12) + 2(10 x 12)

= 2(220) + 2(264) + 2(120)

= 440 + 528 + 240

= 1208 ft squared

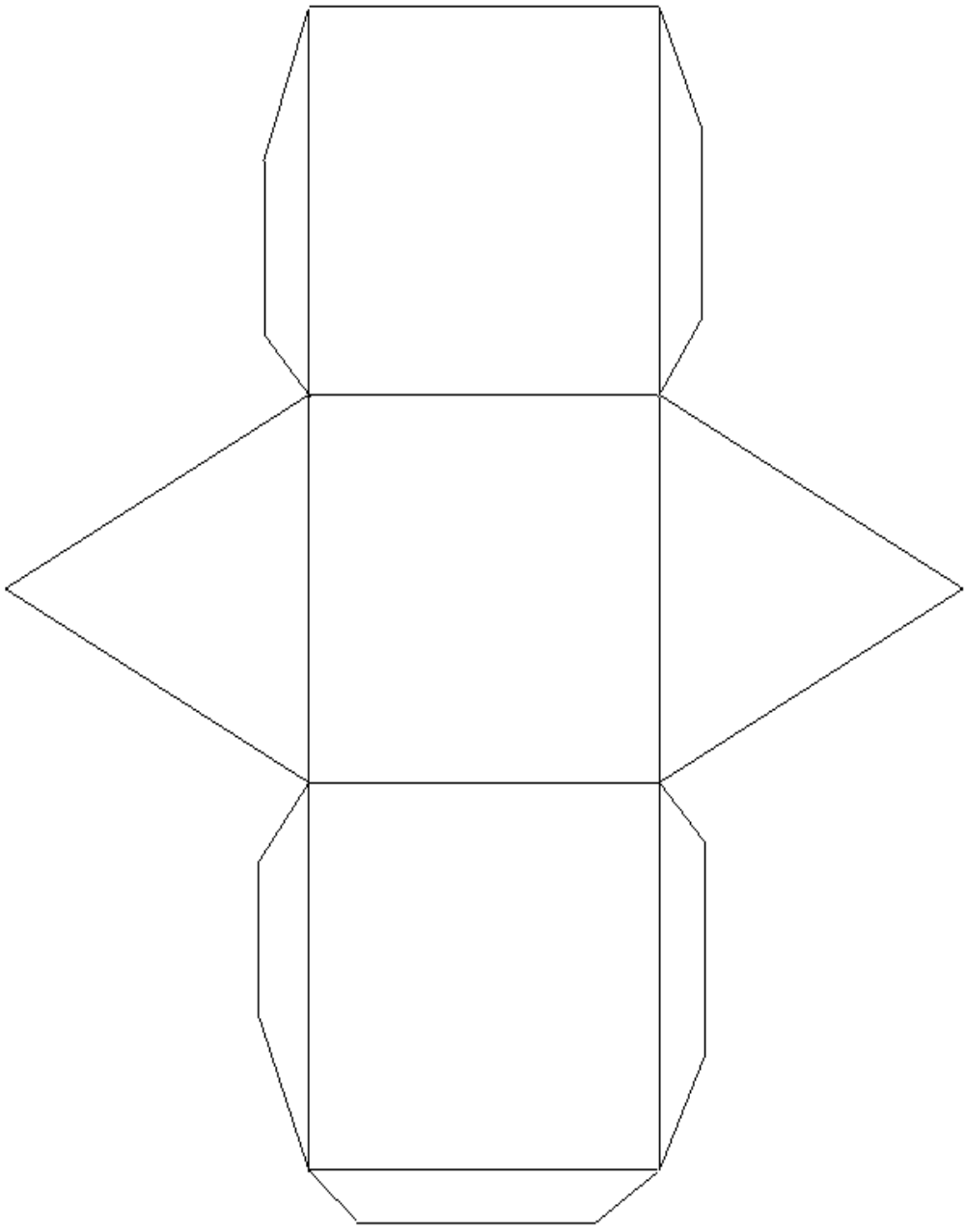
Surface Area of a Triangular Prism



- 2 bases (triangular)
- 3 sides (rectangular)

Unfolded net of a triangular prism

Triangular prism



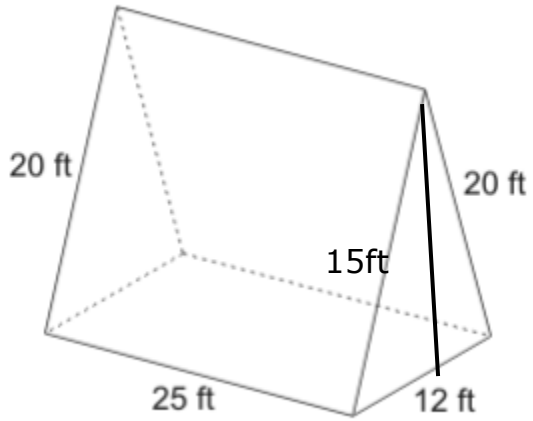
2(area of triangle) + Area of rectangles

Area Triangles = ½ (b x h)

= ½ (12 x 15)

= ½ (180)

= 90



Area Rect. 1 = b x h

= 12 x 25

= 300

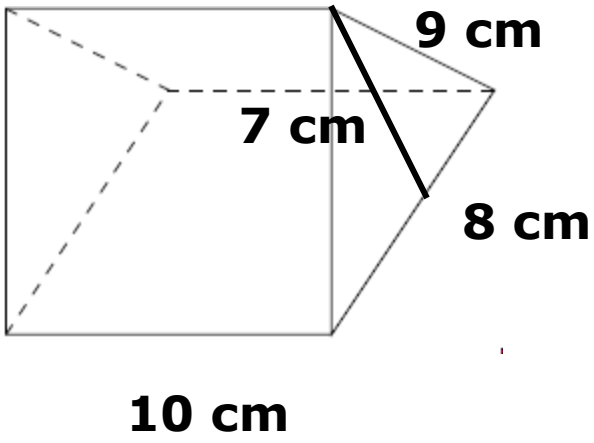
Area Rect. 2 = 25 x 20

= 500

SA = 90 + 90 + 300 + 500 + 500

SA = 1480 ft squared

Practice



Triangles = ½ (b x h)

= ½ (8 x 7)

= ½ (56)

= 28 cm

Rectangle 1 = 10 x 8

= 80 cm

Rectangle 2 = 9 x 10

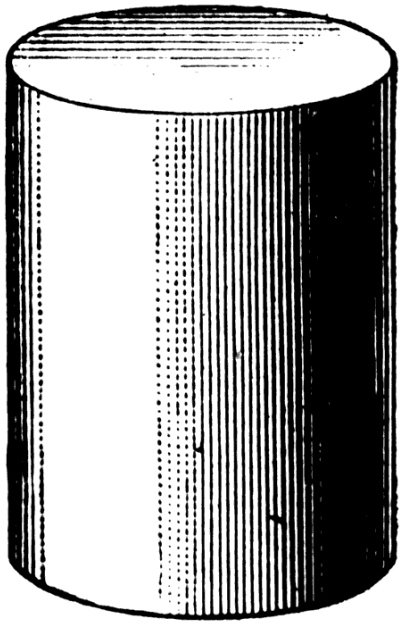
= 90 cm

<u>Add them all up</u>

SA = 28 + 28 + 80 + 90 + 90

SA = 316 cm squared

Surface Area of a Cylinder



Review

- Surface area is like the amount of paper you'll need to wrap the shape.
- You have to "take apart" the shape and figure the area of the parts.
- Then add them together for the Surface Area (SA)

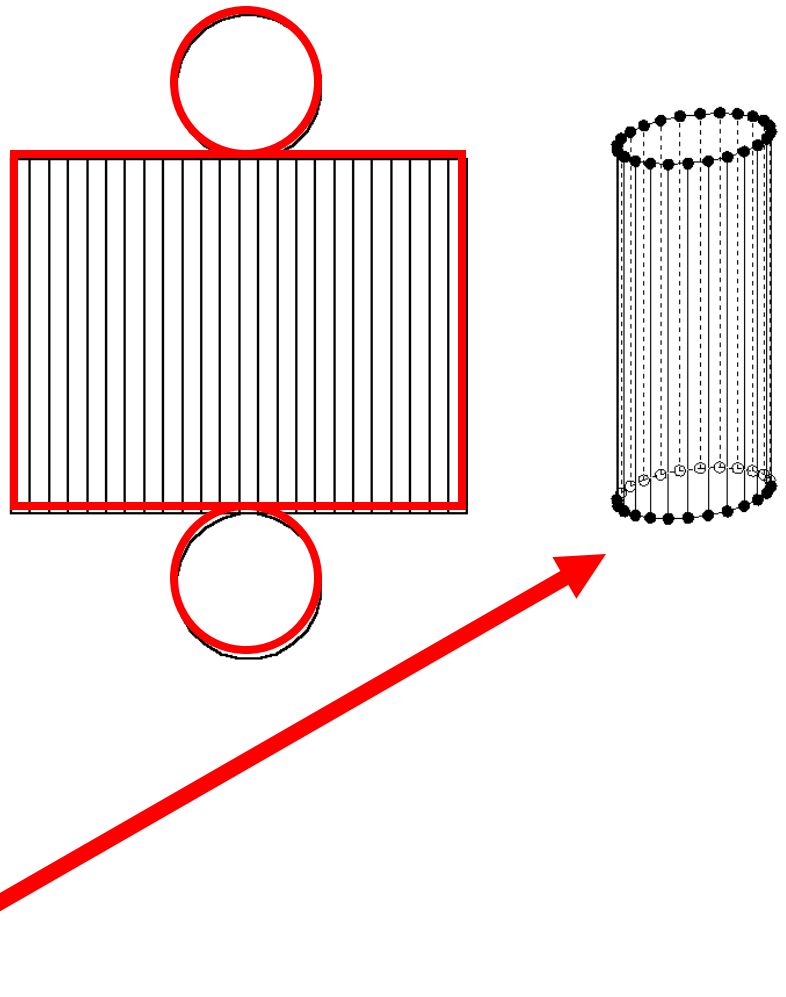
Parts of a cylinder

A cylinder has 2 main parts.

A **rectangle**

and

A **circle** – well, 2 circles really.



Put together they make a cylinder.

The Soup Can

Think of the Cylinder as a soup can.

You have the top and bottom lid (**circles**) and you have the label (a **rectangle** – wrapped around the can).

The lids and the label are related.

The circumference of the lid is the same as the length o the label.

Area of the Circles

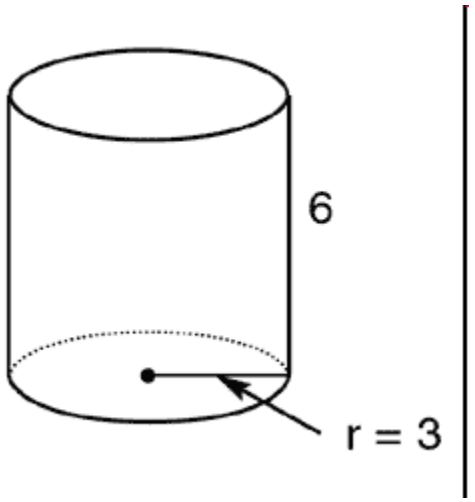
Formula for Area of Circle

$A = \pi r^2$

$= 3.14 \times 3^2$

$= 3.14 \times 9$

$= 28.26$

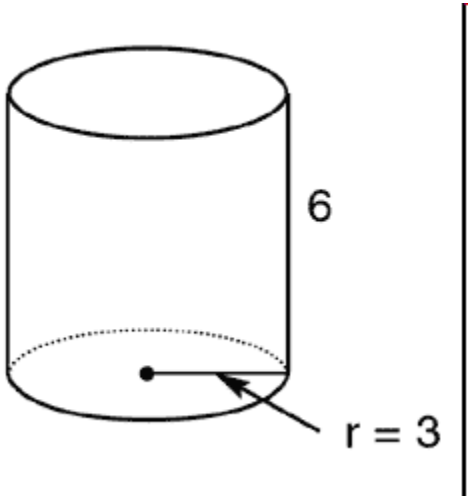


But there are 2 of them so

28.26 x 2 = 56.52 units squared

The Rectangle

This has 2 steps. To find the area we need base and height. Height is given (6) but the base is not as easy.



Notice that the base is the same as the distance around the circle (or the Circumference).

Find Circumference

Formula is

C = π x d

= 3.14 x 6 (radius doubled)

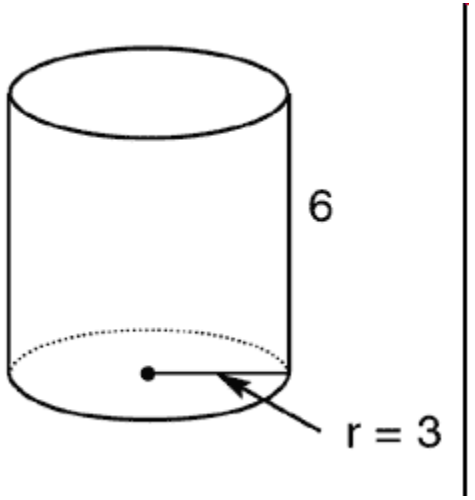
= 18.84

Now use that as your base.

A = b x h

= 18.84 x 6 (the height given)

= 113.04 units squared

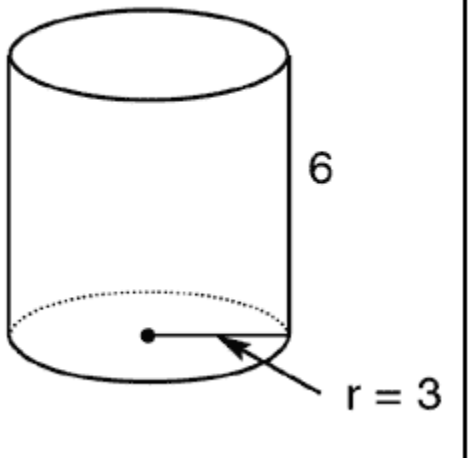


Add them together

Now add the area of the circles and the area of the rectangle together.

56.52 + 113.04 = 169.56 units squared

The total Surface Area!



Formula

$$SA = (\pi d \times h) + 2(\pi r^2)$$

Label — Lids (2)

Area of Rectangle — Area of Circles

Practice

Be sure you know the difference between a radius and a diameter!

$SA = (\pi d \times h) + 2(\pi r^2)$

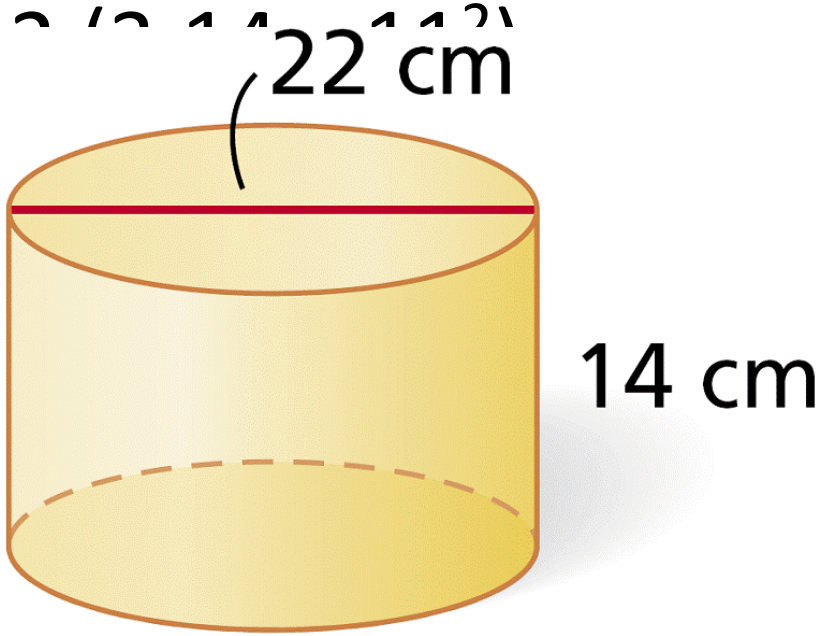
$= (3.14 \times 22 \times 14) +$ [illegible]

$= (367.12) + 2 (3.1$[illegible]

$= (367.12) + 2 (379$[illegible]

$= (367.12) + (759.8$[illegible]

$= 1127\ cm^2$



More Practice!

SA = (π d x h) + 2 (π r^2)

= (3.14 x 11 x 7) + 2 (3.14 x 5.5^2)

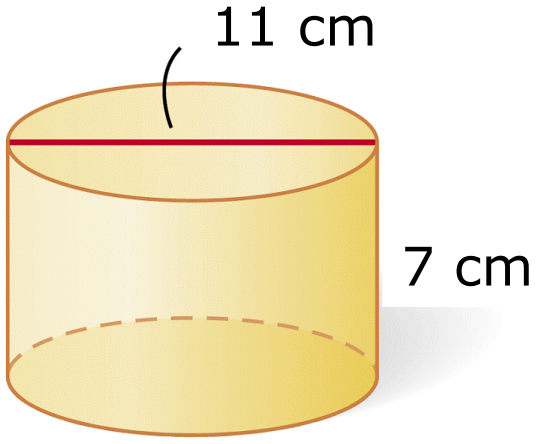
= (241.78) + 2 (3.14 x 30.25)

= (241.78) + 2 (3.14 x 94.99)

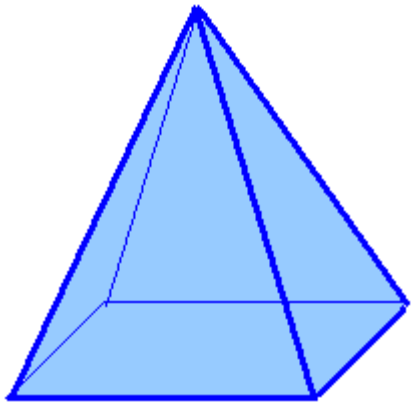
= (241.78) + 2 (298.27)

= (241.78) + (596.54)

= <u>838.32 cm^2</u>



Surface Area of a Pyramid



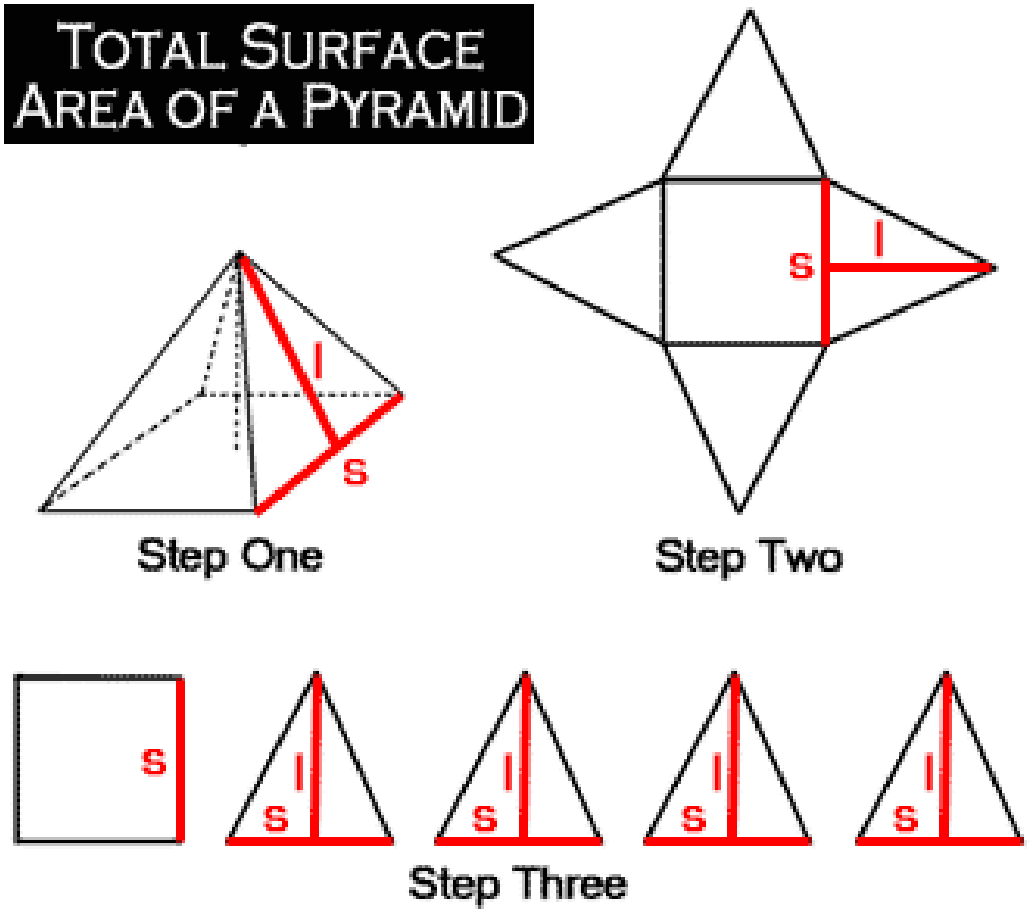
Pyramid Nets

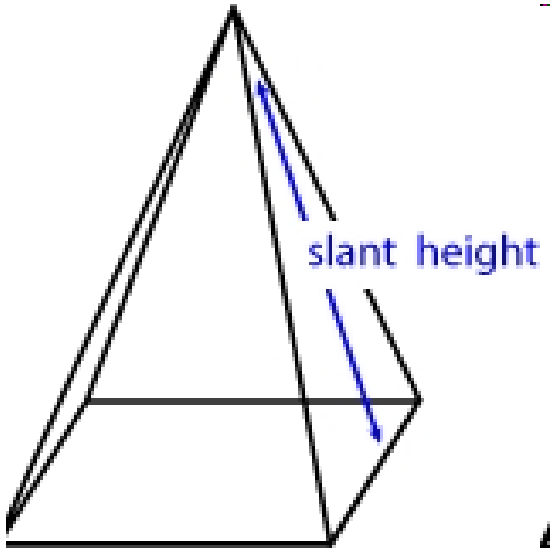
A pyramid has 2 shapes:

One (1) square

&

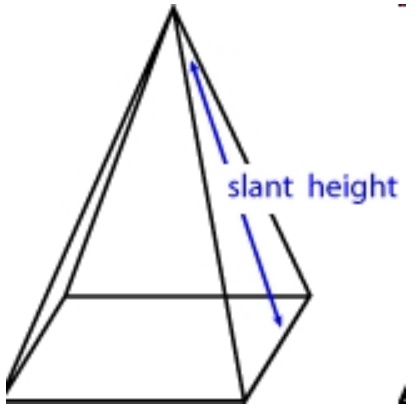
Four (4) triangles





Since you know how to find the areas of those shapes and add them.

Or…



you can use a formula…

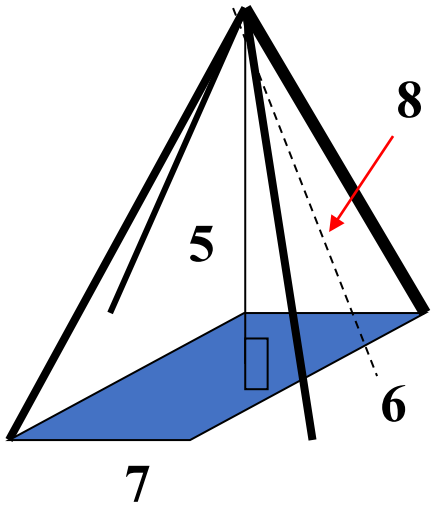
$$\mathbf{SA = \tfrac{1}{2}\, lp + B}$$

Where *l* is the Slant Height and

***p* is the perimeter and**

B is the area of the Base

SA = ½ *lp* + B



Perimeter = (2 x 7) + (2 x 6) = 26

Slant height *l* = 8 ;

SA = ½ *lp* + B

= ½ (8 x 26) + (7 x 6) ***area of the base***

= ½ (208) + (42)

= 104 + 42

= <u>146 units ^{2}</u>

Practice

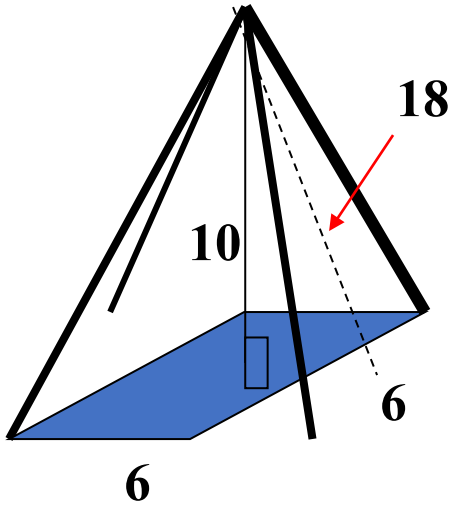
$$SA = \tfrac{1}{2} lp + B$$

$$= \tfrac{1}{2} (18 \times 24) + (6 \times 6)$$

$$= \tfrac{1}{2} (432) + (36)$$

$$= 216 + 36$$

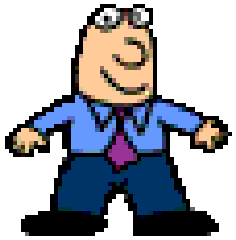
$$= \underline{252 \text{ units}^2}$$



Slant height = 18

Perimeter = 6x4 = 24

What is the extra information in the diagram?



Volume of Prisms and Cylinders

Volume

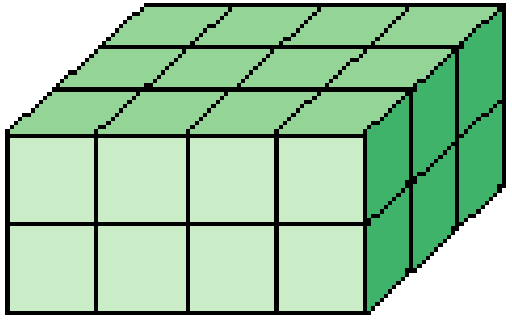
- **The number of cubic units needed to fill the shape. Find the volume of this prism by counting how many cubes tall, long, and wide the prism is and then multiplying.**
- **There are 24 cubes in the prism, so the volume is 24 cubic units.**

2 x 3 x 4 = 24

2 – height

3 – width

4 – length



Formula for Prisms

VOLUME OF A PRISM

The volume *V* of a prism is the area of its base *B* times its height *h*.

$$V = Bh$$

Note – the capital letter stands for the AREA of the BASE not the linear measurement.

Try It

$V = Bh$

Find area of the base

$= (8 \times 4) \times 3$

$= (32) \times 3$

Multiply it by the height

$= \underline{96\ ft^3}$

Practice

$$V = Bh$$
$$= (22 \times 10) \times 12$$
$$= (220) \times 12$$
$$= \underline{2640 \text{ cm}^3}$$

Cylinders

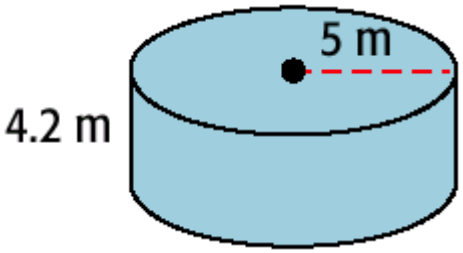
VOLUME OF A CYLINDER

The volume V of a cylinder is the area of its base, πr^2, times its height h.

$$V = \pi r^2 h$$

Notice that πr^2 is the formula for area of a circle.

Try It



$V = \pi r^2 h$

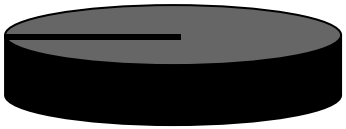
The radius of the cylinder is 5 m, and the height is 4.2 m

$V = 3.14 \cdot 5^2 \cdot 4.2$ Substitute the values you know.

$V = \underline{329.7}$

Practice

13 cm - radius



7 cm - height

$V = \pi r^2 h$ Start with the formula

$V = 3.14 \times 13^2 \times 7$ Substitute what you know

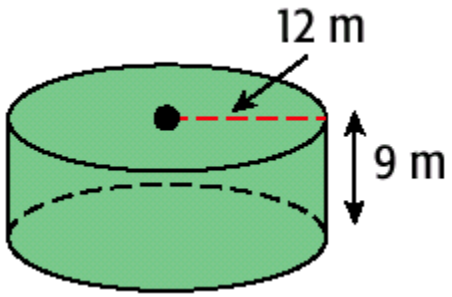
$= 3.14 \times 169 \times 7$ Solve using order of Ops.

$= \underline{3714.62\ cm^3}$

Lesson Quiz

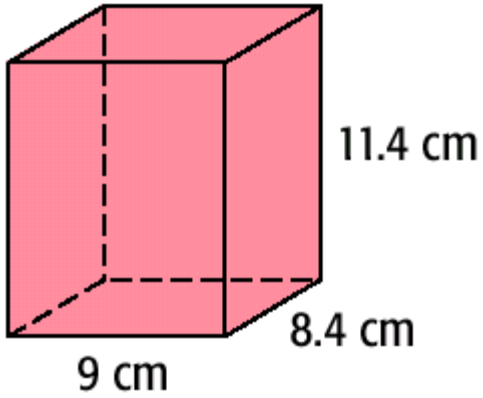
Find the volume of each solid to the nearest tenth. Use 3.14 for π.

1.



4,069.4 m^3

2.

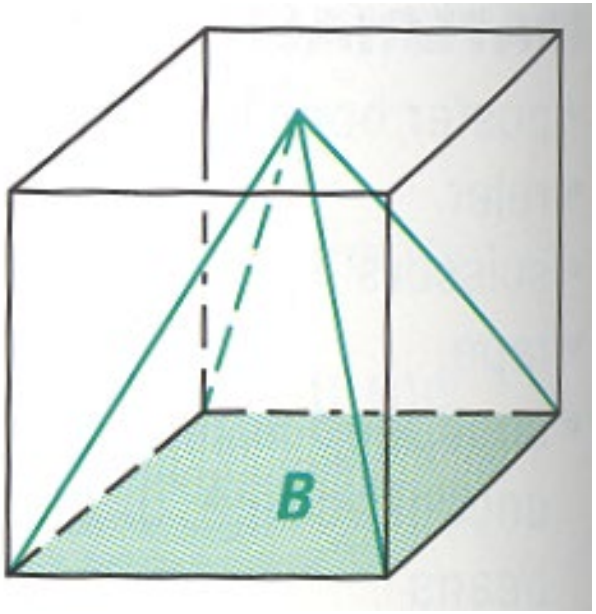


861.8 cm^3

3. triangular prism: base area = 24 ft^2, height = 13 ft

312 ft^3

Volume of Pyramids



Remember that Volume of a Prism is B x h where b is the area of the base.

You can see that Volume of a pyramid will be less than that of a prism.

How much less? Any guesses?

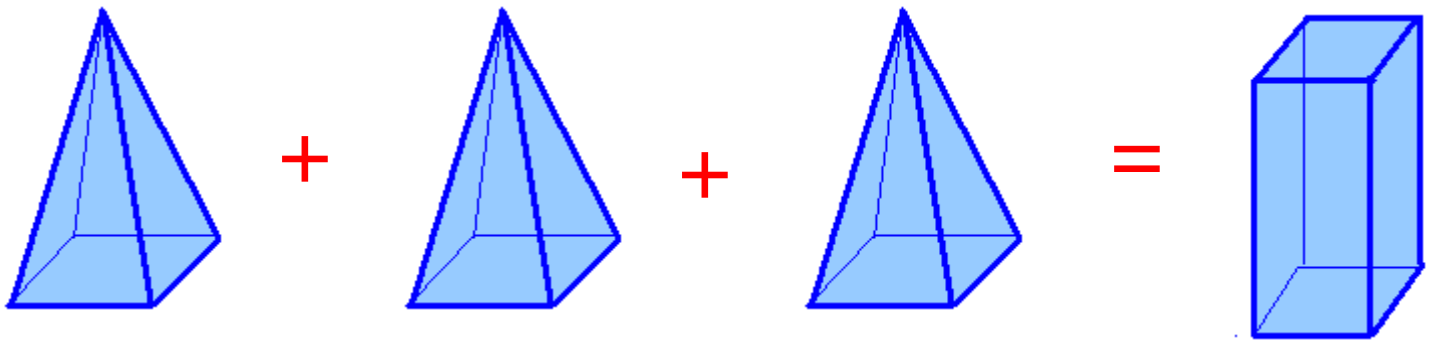
If you said 2/3 less, you win!

Volume of a Pyramid:

V = (1/3) Area of the Base x height

V = (1/3) Bh

Volume of a Pyramid = 1/3 x Volume of a Prism



Find the volume of the square pyramid with base edge length 9 cm and height 14 cm.

The base is a square with a side length of 9 cm, and the height is 14 cm.

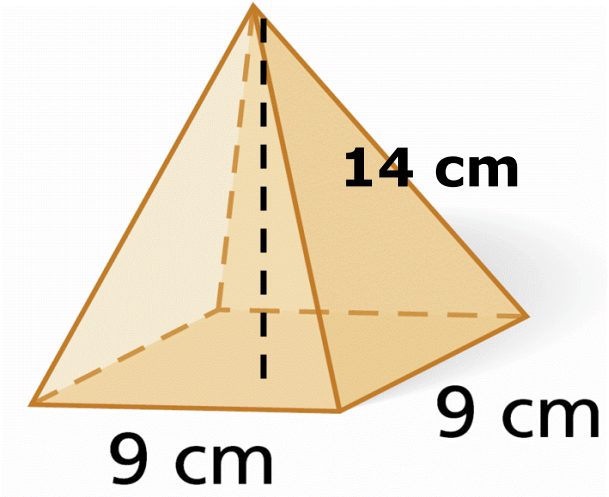
$V = 1/3\ Bh$

$= 1/3\ (9 \times 9)(14)$

$= 1/3\ (81)(14)$

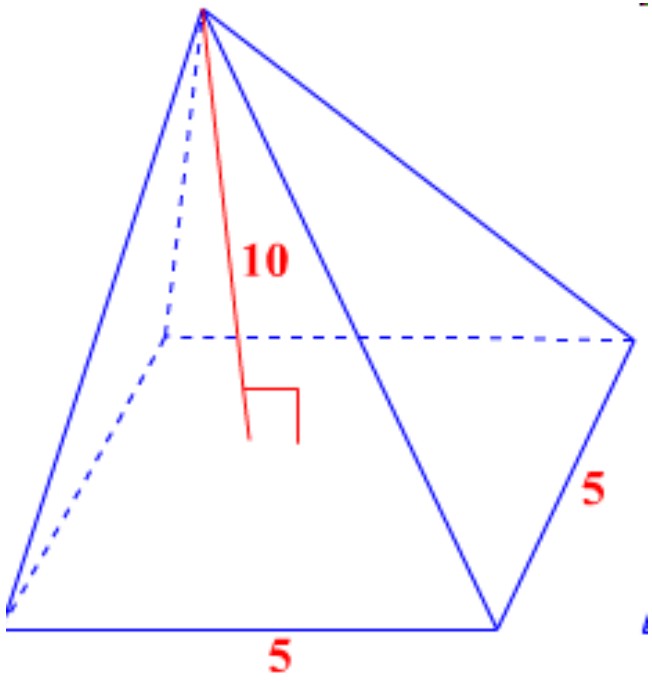
$= 1/3\ (1134)$

$= \underline{378\ cm^3}$



Practice

$$V = 1/3\ Bh$$
$$= 1/3\ (5 \times 5)\ (10)$$
$$= 1/3\ (25)(10)$$
$$= 1/3\ 250$$
$$= \underline{83.33\ \text{units}^3}$$



Quiz

Find the volume of each figure.

1. a rectangular pyramid with length 25 cm, width 17 cm, and height 21 cm

   2975 cm^3

2. a triangular pyramid with base edge length 12 in. a base altitude of 9 in. and height 10 in.

   360 in^3

End